UNIVERSITY OF MARYLAND
Department of Physics
College Park, Maryland

PHYSICS Ph.D. QUALIFYING EXAMINATION
PART I

January 19, 2017 9:00 a.m. – 1:00 p.m.

Do any four problems. Each problem is worth 25 points.
Start each problem on a new sheet of paper (because different faculty members will be grading each problem in parallel).

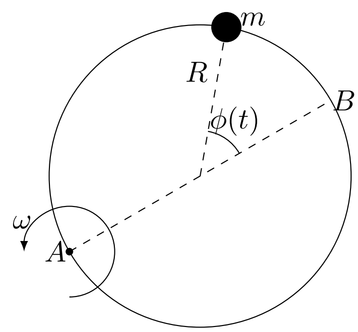
Be sure to write your Qualifier ID ("control number") at the top of each sheet — not your name! — and turn in solutions to four problems only. (If five solutions are turned in, we will only grade # 1 - # 4.)

At the end of the exam, when you are turning in your papers, please fill in a "no answer" placeholder form for the problem that you skipped, so that the grader for that problem will have something from every student.

You may keep this packet with the questions after the exam.

Problem I.1

The figure shows a top-down view of a smooth horizontal circular wire hoop of radius R that is forced to rotate at a fixed angular velocity ω about a vertical axis through the point A. A bead of mass m is threaded on the massless hoop and is free to move around it, with a position specified by the angle $\phi(t)$ that it makes at the center with the diameter AB.



(a) [**5 points**] To get the kinetic energy, first write down the velocity of the bead relative to point A as a function of time t.

(b) [**5 points**] Find the Lagrangian of this system using the ϕ as the generalized coordinate.

(c) [**10 points**] Find the equation of motion for the bead and show that it oscillates around the point B like a simple pendulum.

(d) [**5 points**] Find the frequency of small amplitude oscillations.

Problem I.2

In the Drude model of a conductor, a charge q with mass m is subject to two forces: one from the electric field $\mathbf{E}$ and the other due to frictional drag.

(a) **[8 points]** Suppose the drag can be neglected. Derive an equation for the current density $\mathbf{j}$ from Newton's 2nd law. Then, derive a partial differential equation for the variable $\partial\mathbf{B}/\partial t$ from the Maxwell Equations. Neglect the displacement current. Assume the conductor has n charge carriers per unit volume. We will refer to this system as *a perfect conductor.*

Now, consider a perfect conductor with magnetic permeability $\mu = \mu_0$ in the half-space $x > 0$ and a vacuum for all $x < 0$. Initially, there is zero magnetic field everywhere. At $t = 0$, an external magnetic field is applied in the z-direction such that its magnitude would be $B_0(t)$ everywhere if the perfect conductor were not present.

(b) **[10 points]** Solve your PDE to find $\partial\mathbf{B}(x,t)/\partial t$ and $\mathbf{B}(x,t)$ inside the perfect conductor. Your solutions should be completely specified in terms of the given function $B_0(t)$ and its derivatives.

(c) **[2 points]** Find the magnitude of the current density j inside the conductor.

(d) **[5 points]** A constant B field completely penetrates an ordinary conductor. Now suppose that this ordinary conductor goes through a phase transition to a *perfect* conductor.

(i) Deep inside the conductor, how will the penetrated field evolve in time? Discuss and explain qualitatively. [Hint: use your solution for $(\partial B/\partial t)$ in (b).]

(ii) In this respect, does the perfect conductor behave the same as a superconductor? Why or why not?

Potentially useful: $\nabla \times (\nabla \times \mathbf{A}) = \nabla\nabla \cdot \mathbf{A} - \nabla^2\mathbf{A}$

Problem I.3

Consider a model of a crystal with N regular lattice sites, N interstitial sites, and N identical atoms. In the lowest energy state, all N atoms occupy regular lattice sites. Higher energy configurations are possible in which some of the atoms are displaced to interstitial sites (which need not be close to the vacancies). The excess energy of an atom in an interstitial site is ϵ.

(a) [**6 points**] Compute the number of ways of making n vacancies (and correspondingly filling n interstitial sites) in the lattice. Assume $1 \ll n \ll N$.

(b) [**6 points**] What is the entropy of a state with energy $E = E_0 + n\epsilon$?
Assume $1 \ll n \ll N$, and use an appropriate approximation for $\ln(x!)$.

(c) [**6 points**] Find the average number of interstitial atoms as a function of temperature, $n(T)$, again assuming $1 \ll n \ll N$.

(d) [**4 points**] Using approximate but appropriate values for ϵ and $k_B T$, estimate the fraction of interstitial defects in the atomic lattice of a typical solid at room temperature.

(e) [**3 points**] Under what conditions would the validity of the model break down for a real crystalline solid?

Problem I.4

Set $c = 1$ for this problem.

(a) [**10 points**] A particle with rest mass m_1 and speed v collides with a particle at rest of mass m_2. The collision is completely inelastic, meaning the two particles stick together and no other particle comes out of the collision. Find the rest mass of the composite particle. (Do not assume $v \ll c$.)

(b) [**7 points**] Frame F' has coordinates (t', x'). A fluorescent light bulb in the shape of a long, thin tube is at rest along the x' axis in frame F'. The tube is centered at $x' = 0$ and has length $2L$ in frame F'. At time $t' = 0$ the bulb produces a brief flash of light. Assume the entire length of the bulb lights up simultaneously in F'. (Do not worry about how this is done.) Frame F' moves with constant velocity $\mathbf{v}$ in the x direction with respect to frame F which has coordinates (t, x). The origins of the two frames coincide at time $t' = t = 0$. (i.e. the points $(t, x) = 0$ and $(t', x') = 0$ are the same.) Albert sits at $x = 0$ in frame F. Derive a formula for when Albert sees the two ends of the bulb light up. Make a sketch and give a description of the flash of light as seen by him.

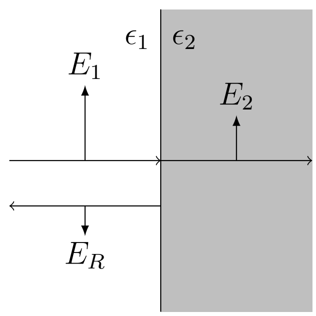
(c) [**8 points**] In frame F' with coordinates (t', x', y', z') a straight rod rotates in the x', y' plane with angular velocity ω' about one of its ends. The fixed end is located at the spatial origin in x' which is $x' = y' = z' = 0$. At time $t' = 0$ the rod lies along the positive x' axis. Frame F' moves with constant velocity $\mathbf{v}$ in the x direction with respect to frame F. The origins of the two frames coincide at time $t' = t = 0$. (i.e. the points $(t, x, y, z) = 0$ and $(t', x', y', z') = 0$ are the same.) Also, there is no spatial rotation between the two frames. Find an equation which gives the shape of the rod in frame F at time $t = 0$. In other words, find an equation for y as a function of x. Given a short (one or two sentence) explanation of why the rod is not straight in F.

Problem I.5

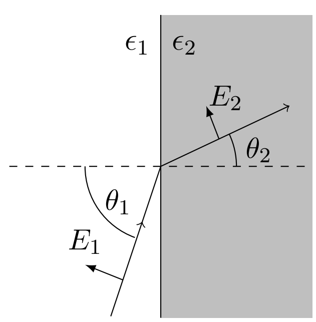
Answer the following questions concerning the propagation of electromagnetic radiation through non-magnetic, dielectric media.

(a) **[5 points]** From Maxwell's equations, derive the boundary conditions for the normal and tangential components of the electric and magnetic fields at the interface between two non-magnetic, dielectric materials with dielectric constants ϵ_1 and ϵ_2.

(b) **[7 points]** An electromagnetic wave with amplitude $\mathbf{E_1}$ propagates normal to the interface, resulting in a reflected wave and transmitted wave of amplitudes $\mathbf{E_R}$ and $\mathbf{E_2}$, respectively, as shown below. Calculate the field reflection coefficient $r = \mathbf{E_R}/\mathbf{E_1}$ and transmission coefficient $t = \mathbf{E_2}/\mathbf{E_1}$.



(c) **[8 points]** An electromagnetic wave propagates at an angle θ_1 with respect to the normal of the interface, with the polarization of $\mathbf{E_1}$ in the plane of incidence as drawn below. The transmitted wave $\mathbf{E_2}$ propagates on the other side at angle θ_2 and we are given that any reflected wave vanishes, or $\mathbf{E_R} = 0$. (There is a particular angle θ_1 where this happens.) Use your result from (a) to determine the angles θ_1 and θ_2 in terms of ϵ_1 and ϵ_2.



(d) **[5 points]** Given the relationship between θ_1 and θ_2 in part (c) above, describe physically why there is no reflected wave. (HINT: consider the reflected wave as radiation from the dipoles in medium 2).