Course Title: PHYS 610 Mathematical Methods and Their Applications in Classical Mechanics and Electrodynamics I

Description for Catalog: This course is the first course of a two-semester graduate level sequence on classical mechanics, electrodynamics and relativity and the mathematics that underlie these subjects. Mathematical methods will generally be introduced in the context of relevant physical problems.

Credits: 4

Topics/Syllabus: (Estimated time on topics in red)

- Calculus of variations and its application in classical dynamics. (4.5 weeks)
  - Introduction to calculus of variations
  - Least action/ Lagrangian formulation of classical mechanics
    - Invariance of formalism under change of variables; constraints
    - very small number of simple examples
    - cyclic variables, symmetry and conservation laws.
      - Example: orbit problems
  - Hamiltonian Formulation
    - Liouville theorem, Poisson brackets
    - Canonical transformations
    - Hamilton-Jacobi theory
    - Action Angle variables
      - Adiabatic Invariants

- Orthogonal Function expansions and solutions to electro- and magneto-statics problems (2 weeks)
  - Rectangular coordinates and Fourier decompositions
    - A few simple examples
  - Spherical coordinates and spherical harmonics
    - Show but do not prove useful properties of spherical harmonics
    - A very few simple examples
  - Cylindrical coordinates and Bessel functions
    - Show but do not prove useful properties of spherical harmonics
    - A very few simple examples

- Green's functions and applications to mechanics and electrostatics (2 weeks)
  - General principle of Green's functions
  - Damped driven oscillator

- Simple application: electrostatics with a plane boundary via method of images for dielectric and/or conductors.
- Green's function in spherical coordinates in free space

- Multipole expansions in electrostatics, magnetostatics (1.5 week)
  - Multipole expansion for static charge distributions
  - Multipole expansion for static current distributions

- Complex Analysis and applications (3 weeks)
  - State but do not prove properties: Analytic functions and maps, branch cuts, contour integration, singularities, residue theorem, Taylor and Laurent series.
  - Stationary phase/steepest descents
  - Conformal mapping and its application to problems in electrostatics

- Other topics as time permits.

Course Title: PHYS 611 Mathematical Methods and Their Applications in Classical Mechanics and Electrodynamics II

Description for Catalog: This course is the second course of a two-semester graduate level sequence on classical mechanics, electrodynamics and relativity and the mathematics that underlie these subjects. Mathematical methods will generally be introduced in the context of relevant physical problems.

Credits: 4

Topics/Syllabus: (Estimated time on topics in red)

- Cartesian tensors and rotations in classical mechanics (highly simplified) (2.5 weeks)
  - Introduction to Cartesian tensors
  - Rotations and Euler angles
  - Moment of inertia tensor and simple examples of rigid body motion.

- Maxwell's equations (3.5 weeks)
  - Maxwell's equation in vacuum
    - Charge conservation
    - Energy density, Poynting vector and conservation of energy
    - Scalar and vector potential, gauge transformations
  - Maxwell's equation in media
    - Remind students of polarization, magnetization E,D, B&H
    - Boundary conditions
  - Wave propagation in media

- Dispersion relations, group and phase velocity
- Complex indices of refraction
- Kramers-Kronig relation

- Radiation; multiple expansion in electrodynamics (2.5 weeks)
    - Retarded Green's functions
    - Fields from oscillating sources; multipole radiation
    - radiation from moving charges (Lenard-Wiechart potentials
    - Thomson scattering

- Relativity (3 weeks)
    - Four vector formulation as encoding time dilatation, relativity of simultaneity etc.
    - Four tensors, the field strength tensor and the covariance of Maxwell's equations.

- Other topics as time permits.

Course Title: PHYS 612 Quantum and Statistical Physics I

Description for Catalog: This course is the first course of a two-semester graduate level sequence on topics in quantum mechanics and statistical mechanics.

Credits: 4

Topics/Syllabus: (Estimated time on topics in red)

- Brief introduction/review of formalism of quantum mmechanics (1.5 weeks)
    - Quantum states as vectors in a Hilbert space, linear algebra, bra & ket vectors, unitary and Hermitian operators, basis and change of basis, Dirac-delta function
    - Postulates of QM (superposition, time evolution as a Unitary operator, Born rule), (non)commuting observables and uncertainty. canonical quantization (from classical to quantum), entanglement

- Elementary applications of Quantum Mechanics (mostly a rapid review) (2.5 weeks)
    - Two level system
    - Free particle
    - 1-d harmonic via raising and lower operator
    - Tunneling and resonance in 1 d
    - Constant mag field Landau levels.

- Quantum mechanics of identical particles (2 weeks)

- Fermions
- Bosons

- Foundations of Statistical Physics (1.5 weeks)
  - Goals of stat. mech./thermodynamics
    - foundations of classical statistical mechanics.
    - Foundations of quantum statistical mechanics Mixed states and density matrices
    - Ensembles: microcanonical, canonical and grand canonical

- Thermo: thermodynamic potentials I (1 week)
  - Brief treatment of Legendre transformations, minimization principles and the relevant thermodynamic potentials.
  - Thermo: Carnot's theorem
- Ideal gases (2.5 weeks)
    - classical
    - Ideal gases: bosonic ideal gasses and Bose-Einstein condensation
    - Ideal gases: fermionic, degeneracy pressure
    - Equipartition theorem
    - gases with internal degrees of freedom, phonons, Einstein/Debye solids
- Phase transitions: Van der Waals, 1st order, critical points, mean field (3 weeks)
  - lattice models, mean field, intro. to universality.

- Other topics as time permits.

Course Title: PHYS 613 Quantum and Statistical Physics II

Description for Catalog: This course is the second course of a two-semester graduate level sequence on topics in quantum mechanics and statistical mechanics.

Credits: 4

Topics/Syllabus: (Estimated time on topics in red)

- Quantum mechanics of rotation and angular momenta (3 weeks)
  - spin ½: Pauli spinors, finite rotations
  - Spherical potential for spinless particles, Spherical harmonics to solve
    - The hydrogen atom
  - Addition of angular momenta, Clebsch-Gordan coefficients.

- Symmetries in quantum mechanics (1.5 weeks)
    - Symmetries, conservation laws and degeneracy.
        - Continuous symmetries
            - Tanslations/ momentum conservation
            - Rotations/angular momentum
        - Discrete symmetries
            - Parity
            - Discrete translations, Bloch's theorem

- Time-independent Perturbation Theory (1 week)
    - formal expansion
    - nondegenerate
        - Examples (only a few)
        - Feynman-Hellmann theorem.
    - degenerate

- Variational methods (.5 weeks)
    - Example Helium

- Time-dependent Perturbation Theory (2 weeks)
    - interaction picture, formal development
    - Fermi's Golden Rule
    - Sudden approximation; Adiabatic approximation
        - Berry Phase.
        - Born-Oppenheimer approximation.

- Scattering (3 weeks)
    - Formal aspects of scattering theory
    - Born Series, Born approximation.
    - partial wave expansion.
    - optical theorem
    - bound states as poles in the T-matrix

- Other Topics as time permits