UNIVERSITY OF MARYLAND
Department of Physics
College Park, Maryland

PHYSICS Ph.D. QUALIFYING EXAMINATION
PART I

August 25, 2016 9:00 a.m. – 1:00 p.m.

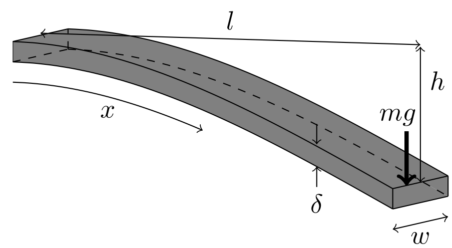
Do any four problems. Each problem is worth 25 points.
Start each problem on a new sheet of paper (because different faculty members will be grading each problem in parallel).

Be sure to write your Qualifier ID ("control number") at the top of each sheet — not your name! — and turn in solutions to four problems only. (If five solutions are turned in, we will only grade # 1 - # 4.)

At the end of the exam, when you are turning in your papers, please fill in a "no answer" placeholder form for the problem that you skipped, so that the grader for that problem will have something from every student.

You may keep this packet with the questions after the exam.

Problem I.1



A diving board consists of a horizontal wooden plank of length l and uniform cross section (width w and thickness δ) that is securely clamped at one end (see diagram). When a diver of mass m stands on the free end of the plank, it deflects downward a distance h with $l \gg h$. Neglect the mass of the plank throughout this problem. In carrying out this problem you will be using the compression properties of the wood in the plank. Specifically, Young's modulus Y gives the relative contraction or expansion of a material $\Delta L/L$ (the strain) in terms of the force per unit area F/A acting on the material. Thus, $\Delta L/L = Y^{-1}F/A$.

(a) [**4 points**] Consider a section of the plank that extends from a distance x from the clamped end to the diver's end of the plank (see diagram). Qualitatively describe all of the forces on this section of the plank, including that from the weight of the diver and the forces associated with compression (below the midplane of the plank) and expansion (above the midplane of the plank). Use the associated torques around the midplane of the plank at the position x to explain how the plank and diver come to equilibrium. Sketch the relevant forces on a diagram of the plank. Estimate the size of the compression/expansion forces F required to balance the torque from the diver.

(b) [**4 points**] To derive an equation for the downward displacement $y(x)$ of the plank at the position x, you will need to know the local radius of curvature R_0 of the bent plank. Show that R_0 is given by

$$\frac{1}{R_0} = \frac{d^2y}{dx^2}.$$

(c) [**7 points**] From the curvature R_0 at position x evaluate the compression of the plank as a function of the distance from the midplane of the plank (along the vertical direction), calculate the force per unit area from Young's modulus and evaluate the torque by integrating the compressive and expansion forces around an axis at the plank's midplane. Balance this torque with that due to the diver to obtain the equation for the plank's displacement $y(x)$, which has the form

$$\frac{d^2y}{dx^2} = G(l - x),$$

where G is a constant that depends on Y, m, g, w, and δ.

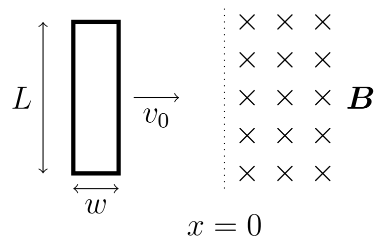
(d) [**4 points**] Integrate the differential equation and use the boundary values $y(0) = 0$, $dy(0)/dx = 0$ to obtain $y(x)$ and an equation for h in terms of G and possibly other quantities from the problem.
Hint: You can answer this question without knowing the expression for G.

Problem continues on next page

(e) [**6 points**] Calculate the period of small amplitude oscillations of the person standing at the end of the plank.
Hint: The effective spring constant for the plank can be obtained by considering the change in h due to a small change in the weight mg of the diver. If you were not able to calculate G, assume that G is some known function of mg.

Problem I.2

Consider an ideal conducting wire (with zero resistance) of radius a and total mass m, shaped into a rectangular loop of length L and width w. Assume $a \ll w \ll L$. The loop has an initial velocity v_0 toward a magnetic half-space with $\boldsymbol{B}$ uniform and into the page for $x > 0$, as shown in the figure. Neglect edge effects.



(a) **[2 points]** Suppose there is no initial current through the wire. Briefly state why the total magnetic flux through the wire must be zero even as it enters the region $x > 0$. What is the direction of the induced current?

(b) **[5 points]** Calculate the inductance of the loop, Λ.

(c) **[5 points]** Calculate the current in the loop as a function of distance x which the loop penetrates the magnetic field. Express your answer in terms of the parameters given above and the inductance Λ.

(d) **[5 points]** Show that the equation of motion of the loop as it penetrates the magnetic half-space can be written in the form

$$\frac{d^2x}{dt^2} = -\alpha^2 x.$$

Express α in terms of the parameters given above.

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(e) **[8 points]** Solve for the penetration distance $x(t)$, given initial velocity v_0 and α. Describe the trajectory, and specify the limits of applicability for this solution.

Problem I.3

Consider a single spinless particle of mass m in a box of volume V and temperature T.

(a) [**10 points**] Calculate the partition function

$$Z_1 = \sum_{\text{states}} e^{-E(\text{state})/k_B T}.$$

Now consider N identical but non-interacting spinless particles at temperature T in a volume V.

(b) [**4 points**] Give the ensemble partition function in terms of Z_1 and use it to evaluate the Helmholtz free energy.

(c) [**8 points**] The entropy of this system is of the form

$$S = k_B \ln(n_0/n) + S_0,$$

where the particle density $n = N/V$. Use the results of part (b) to evaluate n_0 and S_0.

(d) [**3 points**] What are the conditions on density and temperature such that your analysis is valid?

Potentially useful:

$$\int_{-\infty}^{\infty} e^{-ax^2} dx = \sqrt{\frac{\pi}{a}}$$

$$N! \sim \sqrt{2\pi N} N^N e^{-N}$$

Problem I.4

Yuri Milner and Stephen Hawking recently announced $100 million funding for the 'Breakthrough Starshot' program, to accomplish space travel without on-board fuel by shining a laser beam from Earth on the spacecraft.

(a) **[9 points]** Let us first consider the increase in mass of the ship when the photons are completely absorbed by it. We begin with the more general case of a moving particle of rest mass μ colliding with a stationary object of rest mass m_0 and sticking to it. Derive an expression for the resultant mass-squared m^2 as a function of μ, m_0, and ϵ, where $\epsilon = \mu c^2 + K$ is the relativistic energy and K is the kinetic energy of the incoming particle. How does the increase in the rest mass-squared, $m^2 - (m_\mu + m_0)^2$, depend on K?

(b) **[7 points]** Now consider the case when the incoming particle is a photon and let m_0 be the rest mass of the spaceship. Consider the frame in which the spaceship is at rest before absorbing the photon, and has velocity v after absorbing the photon. The mass of the spaceship after it has absorbed the photon becomes $m = \alpha(\beta)m_0$, where $\beta = v/c$. Derive the expression for $\alpha(\beta)$, which involves only the velocity v.

(c) **[2 points]** Explain why adding a mirror to the spaceship would improve the efficiency of this system for accelerating the spacecraft, for a given energy of the incoming photons.

(d) **[7 points]** When the spacecraft is already moving, it can be shown that the 4-momentum transferred by a reflected photon is

$$\boldsymbol{t} = 2\left(\boldsymbol{p}_p - (\boldsymbol{p}_p \cdot \boldsymbol{p}_s)\frac{\boldsymbol{p}_s}{(mc)^2}\right) \tag{1}$$

where $\boldsymbol{p}_p$ and $\boldsymbol{p}_s$ are the 4-momenta (before the reflection) of the photon and the spaceship, respectively. Find the energy transferred by evaluating a suitable component of $\boldsymbol{t}$ in the Earth frame. Then evaluate the "efficiency per photon" of this propulsion system, defined as the ratio

$$\frac{\text{energy gain by spacecraft from reflection of photon}}{\text{energy of photon at incidence}} \tag{2}$$

all energies being measured in the Earth frame. In the limit $\beta \to 1$, this efficiency approaches unity. What is the physical explanation for this?

Possibly useful information: A common form of a momentum 4-vector is $\boldsymbol{p} = (E/c, \vec{p})$ where E and $\vec{p}$ are the total energy and 3-vector momentum of the particle.

Problem I.5

Consider an electromagnetic wave whose electric field $\boldsymbol{E}(z,t)$ is polarized in the x-direction, propagating along the z-direction in a conductor with isotropic conductivity, σ.
Thus $\boldsymbol{E}(z,t) \equiv E_x(z,t)\hat{x}$.

(a) **[6 points]** Derive a wave equation for $E_x(z,t)$ using Maxwell's equations and Ohm's law.

(b) **[4 points]** Assuming that $E_x(z,t)$ is a plane harmonic wave described by

$$E_x = \Re[E_0 e^{i(\omega t - kz)}],$$

derive an expression for the value of k as a function of σ and ω (the dispersion relation). Find the form of k for a good conductor (i.e. neglect the displacement current).

(c) **[5 points]** In a simple model of a conductor, n free electrons per unit volume move in response to an AC electric field $E(\boldsymbol{r})e^{i\omega t}$ and collide with a stationary lattice at a collision rate ν. The resulting relationship between the current density $J(\boldsymbol{r})$ and the electric field $E(\boldsymbol{r})$ is given by $J(\boldsymbol{r}) = \sigma(\omega)E(\boldsymbol{r})$ (Ohm's law). Use this model to determine the constant a in the complex conductivity

$$\sigma(\omega) = \frac{a}{i\omega + \nu}.$$

(d) **[5 points]** For low frequencies $\omega \ll \nu$, calculate an expression for k in part (b) and solve for the attenuation distance for the wave propagating inside a good conductor.

(e) **[5 points]** For high frequencies where $\omega \gg \nu$, calculate an expression for k in part (b). If such a wave were normally incident from vacuum onto a good conductor, what fraction of the power would be reflected?