

PHYS404 (Spring 2024) Introduction to Thermodynamics and Statistical Mechanics

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Lectures: PHY 1201, Tues/Thurs 12.30-1.45pm

Office hours: HM: Tues/Thurs 11.30am-12.30pm, AVW 1415

SS: Tues. 3.30-5pm, PSC 3150

DB: Thurs. 4-5.30pm, PHY 2208

References

F. Reif, *Fundamentals of Statistical and Thermal Physics*, Waveland Press (This is a reprinted version of the original book from the McGraw-Hill series in fundamentals of physics).

Additional reference: C. Kittel and H. Kroemer, *Thermal Physics*, W.H. Freeman & Co.

Topics

Topics will be taken from *Reif*, Chapters 1-10, with sparser selections from Chapters 11-15. The contents section of *Reif* is attached so you can see the topics covered. My class notes will be complete and you should view the book as supplementary. Because I write on the board and do not use PowerPoint, the best way to get the class notes is to come to class.

Problem sets

- Assigned via email approximately every 1½ weeks and handed in at the beginning of class on the due date. There will be roughly 7-8 problem sets over the semester. Some problems will be from Reif and some from a few other sources including me.
- Late submissions of problem sets: If you anticipate your problem set being turned in late, you must email me at least 2 days in advance with an explanation.

Documents and recorded lectures

- Documents, such as problem sets and solutions, will be posted to the ELMS PHYS404 site at the 'Files' tab.
- Any recorded lectures will be posted as mp4 video files at the ELMS PHYS404 site at the 'Panopto Recordings' tab.

Grade breakdown

Problem sets 20%; 2 midterm exams, each 25%; final exam 30%. Improvement is rewarded in this class: Best midterm is counted (you must do BOTH midterms). If the final exam grade is higher than the best midterm, the final exam will count for 80% of the final grade. Grades will be posted to ELMS by the TA/grader.

Exam dates

Unless otherwise specified, all exams will take place in PHY 1201.

Midterm #1- Thurs. March 14

Midterm #2- Thurs. April 30

Final exam- Thursday, May 16, 1:30pm - 3:30pm

contents section of:

F. Reif, Fundamentals of Statistical and Thermal
Physics

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